Formula sheet

1. Basics

Periodic signals

Frequency and period f [Hz], T [s]	$f = \frac{1}{T}$
Angular frequency ω [rad/s]	$\omega = 2\pi \cdot f = \frac{2\pi}{T}$
Sinusoidal signal A – amplitude, φ – initial phase, U_e – offset	$u(t) = A \cdot \sin(\omega \cdot t + \varphi) + U_e$
Initial phase as a time shift φ > 0: the signal shifts to the left	$t_0 = -\frac{\varphi}{\omega}$

Complex numbers

Absolute value	$\lvert z\rvert = \sqrt{a^2 + b^2}$
Addition, subtraction	$z_1 \pm z_2 = (a_1 \pm a_2) + j \cdot (b_1 \pm b_2)$
Multiplication	$z_1 \cdot z_2 = (a_1a_2 - b_1b_2) + j \cdot (a_1b_2 + a_2b_1)$
Division (multiply by the conjugate of the denominator)	$\frac{z_1}{z_2} = \frac{z_1 \cdot z_2^*}{\lvert z_2\rvert^2} = \frac{(a_1a_2 + b_1b_2) + j \cdot (b_1a_2 - a_1b_2)}{a_2^2 + b_2^2}$
Absolute value of a product, quotient	$\lvert z_1 \cdot z_2\rvert = \lvert z_1\rvert \cdot \lvert z_2\rvert, \quad \lvert\frac{z_1}{z_2}\rvert = \frac{\lvert z_1\rvert}{\lvert z_2\rvert}$

2. Electrostatics

Charge, Coulomb's law

Coulomb's law like signs repel, opposite signs attract	$F = k \cdot \frac{\lvert Q_1 \cdot Q_2\rvert}{r^2}$
Coulomb constant	$k = \frac{1}{4\pi \cdot \varepsilon_0} \approx 8.99 \cdot 10^9 \ \frac{N \cdot m^2}{C^2}$
Charge from elementary charges n – number of electrons (degree of ionization)	$Q = n \cdot e$
Number of electrons from mass	$n = \frac{m}{m_e}$

Electric field strength

Definition of field strength E [N/C = V/m]	$E = \frac{F}{q}$

Field of a point charge points away from a positive, towards a negative charge	$E = k \cdot \frac{Q}{r^2}$
Magnitude and direction of the resultant	$\lvert E\rvert = \sqrt{E_x{}^2 + E_y{}^2}, \quad \alpha = \arctan \frac{E_y}{E_x}$
Force on a charge	$F = q \cdot E$

Work, voltage, potential

Voltage U [V = J/C]	$U = \frac{W}{Q}$
Uniform field	$U = E \cdot d$
Work	$W = Q \cdot U = q \cdot E \cdot d$
Voltage between two points moving along the field, the potential decreases	$U_{AB} = U_A - U_B$
Electronvolt	$1\ eV = 1.602 \cdot 10^{-19}\ J$

Capacitor

Capacitance C [F = C/V]	$C = \frac{Q}{U}$
Parallel-plate capacitor	$C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d}$
Stored energy	$W = \frac{1}{2} \cdot C \cdot U^2 = \frac{Q^2}{2C} = \frac{1}{2} \cdot Q \cdot U$
Energy released as the voltage drops	$W = \frac{1}{2} \cdot C \cdot (U_1{}^2 - U_2{}^2) = P \cdot t$
Parallel connection	$C_e = C_1 + C_2 + \ldots$
Series connection	$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$
After connecting (charge conservation)	$U = \frac{C_1U_1 + C_2U_2}{C_1 + C_2}$
Charging with constant current	$\Delta Q = I \cdot \Delta t, \quad \Delta U = \frac{I \cdot \Delta t}{C}$

Constants

Vacuum permittivity	$\varepsilon_0 = 8.854 \cdot 10^{-12}\ \frac{F}{m}$
Elementary charge	$e = 1.602 \cdot 10^{-19}\ C$
Electron mass	$m_e = 9.109 \cdot 10^{-31}\ kg$

3. Resistive networks

Current and charge carriers

Current I [A = C/s]	$I = \frac{Q}{t}, \quad Q = I \cdot t$
Number of electrons passing	$N = \frac{Q}{q_e}$
Current from the motion of carriers n – number of free electrons per unit volume	$I = n \cdot q_e \cdot A \cdot v$
Drift velocity, travel time	$v = \frac{I}{n \cdot q_e \cdot A}, \quad t = \frac{l}{v}$

Resistance

Ohm's law	$R = \frac{U}{I}, \quad U = I \cdot R, \quad I = \frac{U}{R}$
Resistance of a wire ϱ – resistivity, l – length, A – cross-section	$R = \varrho \cdot \frac{l}{A}$
Circular cross-section	$A = \frac{d^2 \cdot \pi}{4}, \quad d = \sqrt{\frac{4A}{\pi}}$
Volume and mass of a wire ϱ_m – density (not the resistivity!)	$V = A \cdot l, \quad m = \varrho_m \cdot V$
Length from mass and resistance	$l = \sqrt{\frac{R \cdot V}{\varrho}}$
Current density J [A/mm²]	$J = \frac{I}{A}$
Temperature dependence for metals $\alpha > 0$ (PTC), for semiconductors, carbon $\alpha < 0$ (NTC)	$R_T = R_{T0} \cdot (1 + \alpha \cdot (T - T_0))$

Power, energy

Power (Joule heat)	$P = U \cdot I = I^2 \cdot R = \frac{U^2}{R}$
Bulb resistance from rated data this is the hot (operating) resistance	$R = \frac{{U_n}^2}{P_n}, \quad I_n = \frac{P_n}{U_n}$
Energy	$W = P \cdot t$
Cable loss, efficiency wire length there and back: $2 \cdot l$	$P_{loss} = I^2 \cdot R_{wire}, \quad \eta \approx 1 - \frac{P_{loss}}{P}$

Equivalent resistance

Series connection	$R_e = R_1 + R_2 + R_3 + \dots$
Parallel connection	$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$

Two resistors in parallel	$R_1 \parallel R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$
n identical resistors	series: $n \cdot R$, parallel: $\frac{R}{n}$

Dividers, series resistor

Voltage divider	$U_1 = U_0 \cdot \frac{R_1}{R_1 + R_2}, \quad U_2 = U_0 \cdot \frac{R_2}{R_1 + R_2}$
Current divider the larger current flows through the smaller resistance	$I_1 = I_0 \cdot \frac{R_2}{R_1 + R_2}, \quad I_2 = I_0 \cdot \frac{R_1}{R_1 + R_2}$
Series resistor U_t – supply voltage, U_n, I_n – rated data of the load	$R_e = \frac{U_t - U_n}{I_n}$

Constants, material properties

Electron charge	$q_e = 1.602 \cdot 10^{-19}\ \mathrm{C}$
Free electrons in copper	$n = 8.5 \cdot 10^{28}\ \frac{1}{\mathrm{m}^3}$
Resistivity of copper	$\varrho_{Cu} = 0.017\ \frac{\Omega \cdot \mathrm{mm}^2}{\mathrm{m}} = 1.7 \cdot 10^{-8}\ \Omega \cdot \mathrm{m}$
Density of copper	$\varrho_m = 8960\ \frac{\mathrm{kg}}{\mathrm{m}^3}$
Temperature coefficient of copper	$\alpha_{Cu} \approx 4.1 \cdot 10^{-3}\ \frac{1}{°\mathrm{C}}$
Conversion	$1\ \mathrm{mm}^2 = 10^{-6}\ \mathrm{m}^2$

4. Thermal calculations

Thermal resistance, chip temperature

Temperature rise R_th [K/W] – thermal resistance	$\Delta T = P_{loss} \cdot R_{th}$
Chip temperature T_j – chip (junction), T_a – ambient	$T_j = T_a + P_{loss} \cdot R_{th}$
Heat transfer at a surface α [W/(m²·K)] – heat transfer coefficient (free air: α ≈ 5), A – cooling area	$R_{th} = \frac{1}{\alpha \cdot A}$

Thermal resistance network with a heatsink

Total thermal resistance (chip → ambient) j – chip, c – case, hs – heatsink, a – ambient; thermal resistances combine like electrical resistances (series: sum, parallel: sum of reciprocals)	$R_{th,e} = R_{jc} + \frac{1}{\frac{1}{R_{chs} + R_{hsa}} + \frac{1}{R_{ca}}}$
Required total thermal resistance	$R_{th,e} = \frac{T_{max} - T_a}{P_{loss}}$

Thermal resistance of the heatsink	$R_{hsa} = \frac{1}{\frac{1}{R_{th,e} - R_{jc}} - \frac{1}{R_{ca}}} - R_{chs}$
Cooling area	$A = \frac{1}{\alpha \cdot R_{hsa}}$

5. RLC networks

RC circuit (DC excitation)

Time constant τ [s]	$\tau = R \cdot C$
Charging: capacitor voltage	$u_C(t) = U_g \cdot (1 - e^{-t/\tau})$
Charging: resistor voltage, current	$u_R(t) = U_g \cdot e^{-t/\tau}, \quad i(t) = \frac{U_g}{R} \cdot e^{-t/\tau}$
Discharging: capacitor voltage	$u_C(t) = U_g \cdot e^{-t/\tau}$
Discharging: resistor voltage, current	$u_R(t) = -U_g \cdot e^{-t/\tau}, \quad i(t) = -\frac{U_g}{R} \cdot e^{-t/\tau}$
Characteristic values (charging, $t = \tau$)	$u_C(\tau) \approx 0.63 \cdot U_g, \quad i(\tau) \approx 0.37 \cdot \frac{U_g}{R}$
Settling time (1 % error) $e^{-t/\tau} = 0.01$	$t \approx 4.6 \cdot \tau$
Time to a given capacitor voltage	$t = -\tau \cdot \ln(1 - \frac{u_C}{U_g})$

RL circuit (DC excitation)

Time constant τ [s]	$\tau = \frac{L}{R}$
Switch-on: inductor voltage	$u_L(t) = U_g \cdot e^{-t/\tau}$
Switch-on: resistor voltage, current	$u_R(t) = U_g \cdot (1 - e^{-t/\tau}), \quad i(t) = \frac{U_g}{R} \cdot (1 - e^{-t/\tau})$
Switch-off: inductor voltage	$u_L(t) = -U_g \cdot e^{-t/\tau}$
Switch-off: resistor voltage, current	$u_R(t) = U_g \cdot e^{-t/\tau}, \quad i(t) = \frac{U_g}{R} \cdot e^{-t/\tau}$

AC: reactance, impedance

Capacitive reactance f increases → X_C decreases	$X_C = \frac{1}{\omega \cdot C} = \frac{1}{2\pi \cdot f \cdot C}$
Inductive reactance f increases → X_L increases	$X_L = \omega \cdot L = 2\pi \cdot f \cdot L$
Impedance (series circuit) $X = X_C$ (RC circuit) or $X = X_L$ (RL circuit); reactance and resistance add as vectors	$\lvert Z \rvert = \sqrt{R^2 + X^2}$

Current and voltages	$I = \frac{U_0}{\lvert Z \rvert}, \quad U_R = R \cdot I, \quad U_X = X \cdot I$
Generator voltage the voltages are at 90° to each other, so they do not add arithmetically	$U_0 = \sqrt{U_R^2 + U_X^2}$